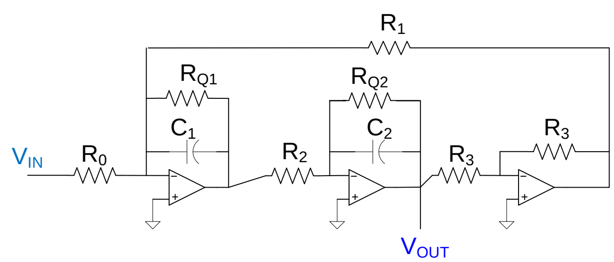
EE 508
Mid-term Exam
Fall 2024
Due Wednesday Oct 29

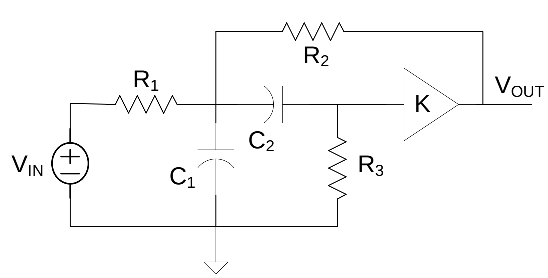
Problem 1 Consider the two integrator loop filter shown below. Assume the degrees of freedom are constrained by setting $C_1=C_2=C$, $R_0=R_2=R_3$, $R_{Q1}=R_{Q2}=R_Q$.

a) Determine the transfer function $T(s)=V_{OUT}/V_{IN}$
b) Derive an expression for the pole Q
c) Derive an expression for $S_{R_{Q1}}^{Q}$
d) Compare the performance of this structure with the more standard structure where everything is the same except the second integrator is lossless (i.e. RQ2=∞).



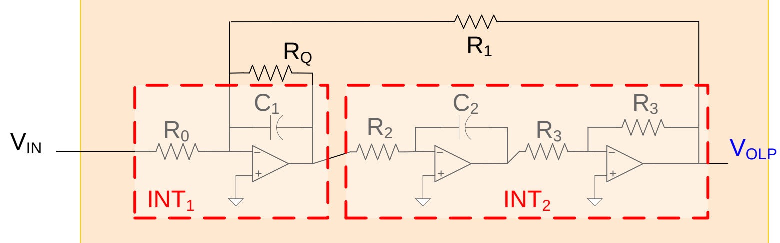
Problem 2 The standard +KRC biquad is shown below. Two specific implementations in which the extra degrees of freedom were used were considered. Type 1 was where R1=R2=R3=R and C1=C2=C. Type 2 was where $R_1=R_2=R_3=R$ and K=1. A significant difference in sensitivities were observed.

a) Derive an expression for the capacitor ratio,C2/C1, denoted as θ, as a function of the pole Q for the Type 2 realization.
b) A transitional Type1:Type2 realization, denoted at a Type 3 structure could be obtained by setting R1=R2=R3=R and C2/C1=γ where 1<γ<θ. When γ=1 this becomes a Type 1 structure and when γ=θ this becomes a Type 2 structure. But for intermediate values of γ the properties will be different than either of the parent types. Compare the sensitivities of the Type 3 structure with the other two for Q=10.
c) Can you identify any performance benefits by relaxing the requirement that R1=R2=R3=R.



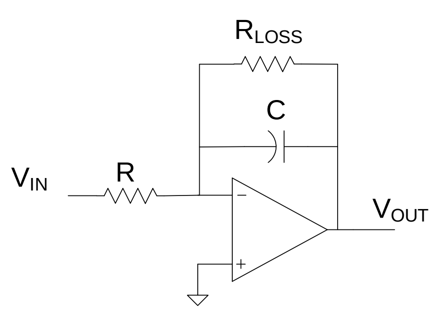
Problem 3 Consider the standard two integrator loop filter shown below with the standard assumption of $R_0=R_1=R_2=R$ and $C_1=C_2$.

a) Determine the sensitivity of the amplifier pole with respect to R1
b) Repeat part a) but determine the sensitivity with respect to C2
c) Determine the pole sensitivity with respect to $\tau=1/GB$ of the middle operational amplifier.



A Popular Second-Order Lowpass Filter

Problem 4 A standard Miller integrator is shown below. One of the major challenges of integrating this circuit for audio frequency applications is the area required for the resistors and the capacitor. In particular, if the loss is small, the resistor RLOSS can be very large. If a standard unit cell is used for the resistor R and if a number of these are placed in series to form RLOSS, then the area for RLOSS can be much larger than the area for R. And, even if there is no loss in the integrator, the area required for the resistor R is often unacceptably large. For example, if in a process the sheet resistance is $R_\square=30\Omega/\square$, and the resistance require to achieve a given integrator unity gain frequency is determined to be R=30K, and if RLOSS=20R, the number of squares of resistance is given by n=n1+n2 where n1 is the number of squares for R and n2 is the number of squares for RLOSS. In this example, n1=1000 and n2=20,000 so the total number of squares is n=21,000.



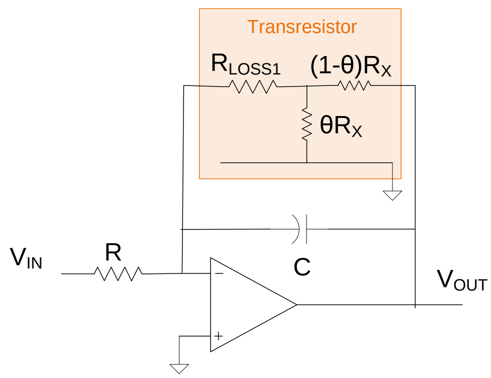
Standard Lossy Miller Integrator

A lossy integrator with a transresistive loss element is shown below. Also shown below is a lossy integrator with two transresistive elements.

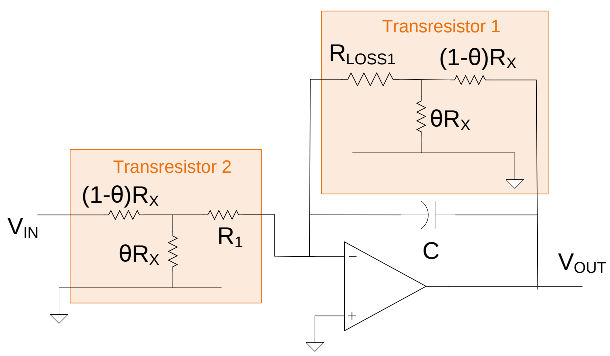
a) Derive the transfer function for the circuit with a single transresistor under the assumption $\theta R_X << R_{LOSS1}$ and show that for an appropriate R_{LOSS1} and θ, this lossy integrator can have the same integrator gain characteristics as the Standard Lossy Miller Integrator.
b) If $R_X=R$ (where R is the input resistance in the Standard Lossy Miller Integrator)and $\theta=0.1$, compare the total area of the resistors (specified as the total number of squares)required to realize the same integrator unity gain frequency and same loss as was obtained with the Lossy Miller Integrator. Assume the input still drives a resistor of value R.

c) Determine the total resistor area (specified as the total number of squares) required for the lossy integrator with the two transresistance elements if R_X=R and θ=0.1 if this structure has the same loss and the same integrator unity gain frequency as the Lossy Miller Integrator. In this derivation, R is the input resistance of the original Miller Lossy Integrator.

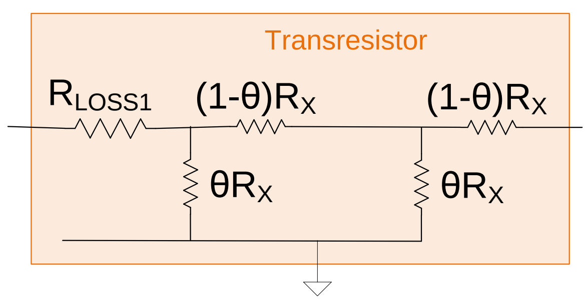
d) This process can be repeated again to further reduce the resistor area by using a two-stage transresistor ladder shown below. Is this use of transresistor elements (with one, two, or even more stages) an effective method for reducing the area requirements for building integrated audio frequency or even sub-audio frequency active filters? Discuss the benefits and/or limitations of this approach.



Lossy Miller Integrator with Transresistor Loss Element



Lossy Miller Integrator with Two Transresistor Elements



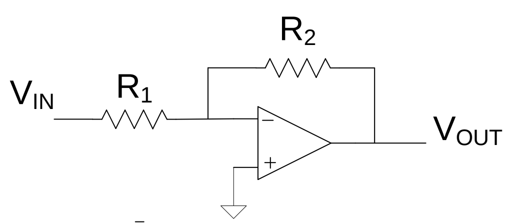
Transresisor with Two-Stage Ladder

Problem 5 The integrator in US Patent 8,436,679 uses an R-2R ladder instead of a single resistor resulting in a unity gain frequency that is digitally programmable. Compare the performance of this R-2R integrator with that of the basic Miller integrator.

Problem 6 A new method of building filters is proposed in US Patent 8,952,749 by Mediatek. Obtain the transfer function for the basic embodiment in this patent which is included in Fig. 2 and comment on the practicality of this circuit.

Problem 7 Consider the following amplifier where the op amp is assumed to be ideal. Assume the amplifier is to be designed to have a gain of $16 \pm 0.5\%$ and the layout of the resistors uses a common-centroid geometry to cancel gradient effects. Assume the resistors are created by using a series connection of a unit resistors of 50 ohms and area 5um2. Assume the matching characteristics for closely-placed interdigitized resistors is characterized by the Pelgrom parameter XR=.01μ-1.

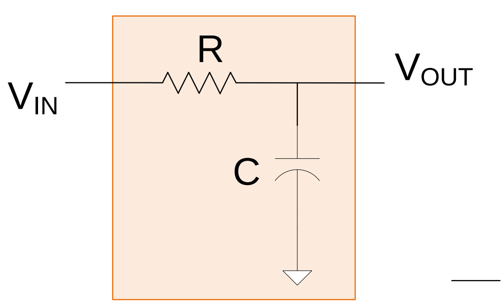
a) Determine the area for the resistors and the yield if the nominal value of R2 is 3.2K.
b) How does the area and yield change if the nominal value of R2 is increased to 32K



Problem 8 Consider the first-order RC filter shown below. Assume the resistor is comprised of a series connection of resistors with resistance density D_R and the capacitor has a capacitance density of D_C. Assume also that ω_0 is the 3B frequency.

The area required to realize this filter is given by $A=A_R+A_C$ where A_R is the area for the resistor and A_C is the area for the capacitor.

a) Give an expression for the area required to realize this filter in terms ω_0, D_R, and D_C.
b) If the value of R and C are selected to minimize the total area for a given ω_0, how does A_R compare to A_C?



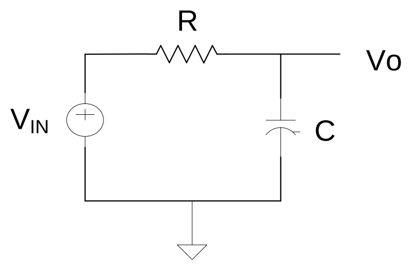
Problem 9 Consider now a requirement where two of the filters considered in the previous problem are to be designed and that the poles of these two filters are to be closely matched and are to be nominally equal to ω_0. Assume that a common-centroid layout is used for the resistors and a common

centroid layout is used for the capacitors to eliminate gradient effects. Assume the standard deviation of a resistor and of a capacitor are given respectively by the equations

$$\sigma_R = \frac{X_R}{\sqrt{A_R}} \qquad \sigma_C = \frac{X_C}{\sqrt{A_C}}$$

where the parameters X_R and X_C are constants characteristic of the process and where A_R and A_C are the areas of the resistor and capacitor respectively. Following the notation of the previous problem, the resistance density and the capacitance density are respectively D_R and D_C.

a) Determine the relative area of the resistor and the capacitor, A_R and A_C to satisfy the nominal ω_0 requirement that will minimize the variance of the difference between the two band edges, ω_{01} and ω_{02} .
b) X_R=.01u^{-1} and X_C=.005u^{-1}?
c) How much would the variance increase if the sizing strategy to achieve minimum area were used instead of the sizing strategy to minimize variance? Assume the same values for X_R and X_C used in part b).



Problem 10 Obtain the lowest-order approximating function for a Chebyschev filter that meets the filter mask requirements given below. Verify your solution with a plot of the transfer function magnitude.